CAFPE-39/04
UB-ECM-PF-04-27
UG-FT-169/04

Charged Kaon $K \to 3\pi$ CP Violating Asymmetries [1]

Elvira Gámiz[a)], Joaquim Prades[b)] and Ignazio Scimemi[c)]

[a)] Department of Physics & Astronomy, University of Glasgow
Glasgow G12 8QQ, United Kingdom.

[b)] Centro Andaluz de Física de las Partículas Elementales (CAFPE) and
Departamento de Física Teórica y del Cosmos, Universidad de Granada
Campus de Fuente Nueva, E-18002 Granada, Spain.

[c)] Facultat de Física-ECM, Universitat de Barcelona
Av. Diagonal 647, E-08028 Barcelona, Spain.

Abstract

First full next-to-leading order analytical results in Chiral Perturbation Theory for the charged Kaon $K \to 3\pi$ slope g and decay rates CP-violating asymmetries are presented. We discuss the constraints that a measurement of these asymmetries would impose on the Standard Model calculations of ε'_K and the kind of information it can provide on Im G_8, Im $(e^2 G_E)$ and higher order weak couplings.

October 2004

[1]Invited talk given by J.P. at " 32nd ICHEP, 16-22 August 2004, Beijing, China".

1 Introduction and Motivation

Direct CP violation has been established unambiguously in $K \to \pi\pi$ decays by KTeV[1] and NA48[2] through the measurement of $\mathrm{Re}(\varepsilon'_K/\varepsilon_K)$. Its present world average is[1, 2, 3, 4]

$$\mathrm{Re}\left(\frac{\varepsilon'_K}{\varepsilon_K}\right) = (1.67 \pm 0.16) \cdot 10^{-4}\,. \tag{1}$$

The theoretical understanding of this quantity within the Standard Model (SM) is not at the same level. We mention here just the most recent advances: the Chiral Perturbation Theory (CHPT) calculation[5, 6] and the isospin breaking corrections[7] have both fully been done at next-to-leading order (NLO) and the rôle of Final State Interactions (FSI) has also been understood[8] –for a more extensive description of these works and references, see[9]. There have been also recent advances on the calculation of the leading-order (LO) CHPT couplings Im G_8 and Im $(e^2 G_E)$ [10, 11, 12, 13] –they are not fully under control though and more work is still needed.

Asymmetries in the Dalitz variable slope g of $K \to 3\pi$ amplitudes are another very promising place to study direct CP violation in Kaon decays. In fact, there are several experiments, NA48/2[14] at CERN, KLOE[15] at Frascati and OKA[16] at Protvino, that have announced an expected sensitivity to these asymmetries of the order of 10^{-4}, one order of magnitude better than at present[17]. On the theory side, though the first calculation of $K \to 3\pi$ at NLO in CHPT was done long ago[5], the analytical full results were unfortunately not available until recently[18]. The CP asymmetries were therefore predicted just at LO plus various estimates of NLO effects[19]. The first full NLO calculation within CHPT for those asymmetries was done in [20]. Here, we report results just for Δg_C –results for the rest of the asymmetries can be found.

2 Technique

The effective quantum field theory of the SM at energies below or of the order of 1 GeV is CHPT[21]. Some introductory lectures on CHPT can be found in [22] and recent reviews in[23].

The full one-loop calculation in the isospin limit was done in [18, 20] and they both fully agree. All the needed notation and definitions were given there. Recently, some isospin breaking corrections have also been calculated[24]. Notice that some misprints in the first reference in[20] were reported in the third reference in[20].

At this order there appear eleven unknown counterterms. The real part of them and of the LO couplings G_8 and G_{27} can be fixed from a fit to all available $K \to \pi\pi$ amplitudes at NLO in CHPT[6] and $K \to 3\pi$ amplitudes and slopes

also at NLO[18, 20]. This was done in [18] and we used them as inputs in all the results we report here.

The values we used for Im $(e^2 G_E)$ and Im G_8 can be found in[20]. They are taken mainly from [10, 11] but are also compatible with [12, 13].

The imaginary part of the order p^4 counterterms, Im $\widetilde{K}_i$, is much more problematic. They cannot be obtained from data and there is no available calculation for them at NLO in $1/N_c$. One can still get the order of magnitude and/or signs of Im $\widetilde{K}_i$ using several approaches. We followed[20] a more naive approach that is enough for the purpose of estimating the effect of those counterterms. We assumed that the ratio of the real to the imaginary part is dominated by the same strong dynamics at LO and NLO in CHPT, namely

$$\begin{aligned} \frac{\mathrm{Im}\,\widetilde{K}_i}{\mathrm{Re}\,\widetilde{K}_i} &\simeq \frac{\mathrm{Im}\,G_8}{\mathrm{Re}\,G_8} \simeq \frac{\mathrm{Im}\,G_8'}{\mathrm{Re}\,G_8'} \simeq (0.9 \pm 0.3)\,\mathrm{Im}\,\tau \\ &= -(0.9 \pm 0.3)\ \mathrm{Im}\left(\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}\right). \end{aligned} \tag{2}$$

3 $K \to 3\pi$ CP Violating Asymmetries

The definition of the CP-violating asymmetries in the slope g and analogous asymmetries for the decay rates Γ can be found, for instance, in[20]. They start at $\mathcal{O}(p^2)$ in CHPT and at NLO require the FSI phases of three-pions at NLO, i.e. an $\mathcal{O}(p^6)$ calculation.

Though the full result is unavailable at present, we have calculated analytically the expected dominant part which comes from two-bubble diagrams[20]. Including these and substituting the pion and Kaon masses, Re G_8, G_{27} and the real part of the NLO CHPT couplings, the result we get for Δg_C is

$$\begin{aligned} \frac{\Delta g_C}{10^{-2}} &\simeq \Big[(0.7 \pm 0.1)\,\mathrm{Im}G_8 + (4.3 \pm 1.6)\,\mathrm{Im}\widetilde{K}_2 \\ &- (18.1 \pm 2.2)\mathrm{Im}\widetilde{K}_3 - (0.07 \pm 0.02)\mathrm{Im}(e^2 G_E)\Big]\,. \end{aligned} \tag{3}$$

When values for the imaginary part of the needed couplings are taken as explained in the previous section, on gets

$$\Delta g_C = -(2.4 \pm 1.2) \cdot 10^{-5}\,. \tag{4}$$

Results for the rest of the asymmetries can be found in[20]

4 ε'_K vs $K \to 3\pi$ CP Violating Asymmetries

Including FSI to all orders, CHPT and isospin breaking at NLO[6, 7, 8], one gets

$$\frac{\varepsilon'_K}{\varepsilon_K} \simeq -\Big[(1.88 \pm 1.0)\,\mathrm{Im}G_8 + (0.38 \pm 0.13)\,\mathrm{Im}(e^2 G_E)\Big]\,. \tag{5}$$

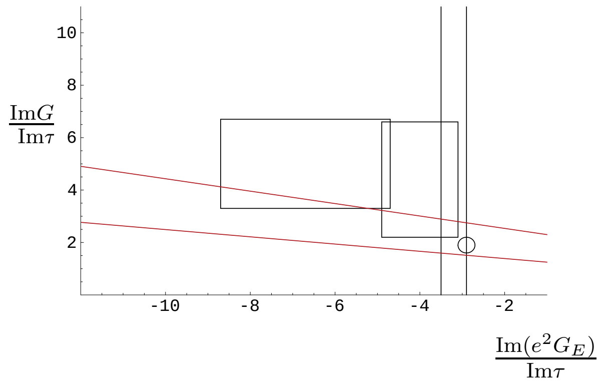

Figure 1: ε'_K: Theory vs Experiment. See text for explanation.

Using this result, the experimental one in (1) imposes that Im G_8 and Im (e^2G_E) are constrained to be within the horizontal band in Figure 1. Also plotted in the same figure are the predictions for those couplings from [10, 11] –rectangle on the right–, from [12] –rectangle on the left– and from [13] –vertical lines.

A measurement of Δg_C can have an important impact on constraining what we know on Im G_8 and and Im (e^2G_E) from ε'_K. To assess the quality of these constraints, we plot in Figure 2 the comparison between what one gets with ε'_K, the theory predictions and the dashed horizontal band that one gets using (3) for $\Delta g_C = -3.5 \cdot 10^{-5}$. In Figure 3, we show the same plots for $\Delta g_C = -1 \cdot 10^{-5}$.

5 Conclusions

The CP violating asymmetry Δg_C is dominated by the value of Im G_8 and its final uncertainty is mainly from this input. This is the only asymmetry with an uncertainty smaller than 50%. The predictions for the rest of CP asymmetries can be found in [20].

The eventual measurement of Δg_C will then provide a check of consistency with ε'_K –see Figures 2 and 3. The SM prefers values for this asymmetry larger than $-0.4 \cdot 10^{-4}$ and an experimental result of the order or smaller than $-2 \cdot 10^{-4}$ would indicate the presence of new physics. For a discussion on possible SUSY implications of a measurement of these asymmetries see [25].

The CP asymmetries Δg_N and in the decay rates were also discussed in [20] and we found that they are dominated by the imaginary part of the $\mathcal{O}(p^4)$ counterterms. A measurement of these asymmetries would therefore give very inter-

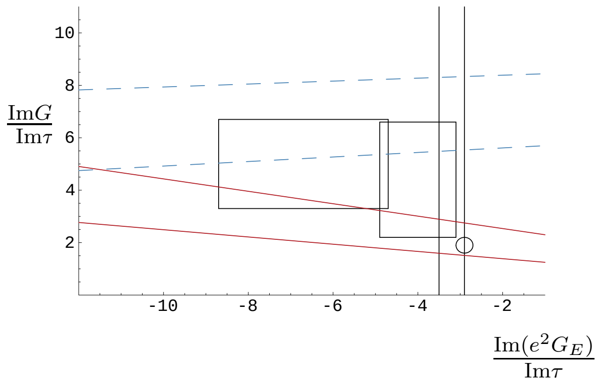

Figure 2: ε'_K vs Δg_C for $\Delta g_C = -3.5 \cdot 10^{-5}$.

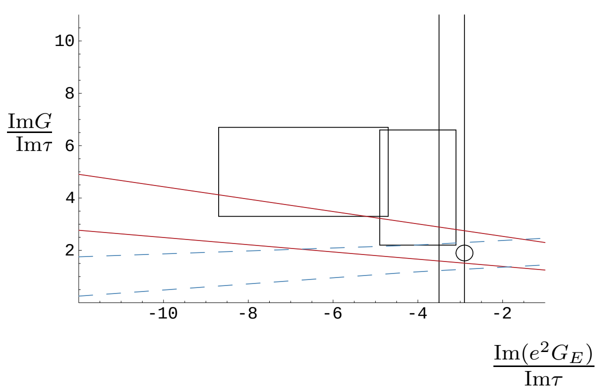

Figure 3: ε'_K vs Δg_C for $\Delta g_C = -1 \cdot 10^{-5}$.

esting information on the size of the imaginary parts of those couplings.

As a general conclusion, direct CP violating asymmetries in $K \to 3\pi$ provide extremely interesting and valuable information on the SM which is complementary to the one obtained from ε'_K. We are therefore eagerly awaiting the new experimental results!

Acknowledgments

E.G. is indebted to the EU for a Marie Curie Fellowship. This work has been supported in part by the EU RTN Network EURIDICE under Contract No. HPRN-CT2002-00311 (I.S. and J.P.), by MEC (Spain) and FEDER (EU) Grants No. FPA2001-03598 (I.S.) and FPA2003-09298-C02-01 (J.P.) and by Junta de Andalucía Grant No. FQM-101 (J.P.).

References

[1] KTeV Collaboration, *Phys. Rev.* D **67** (2003) 012005;
Phys. Rev. Lett. **83** (1999) 22.

[2] NA48 Collaboration, *Phys. Lett.* B **544** (2002) 97; ibid. **465** (1999) 335; *Eur. Phys. J.* C **22** (2001) 231.

[3] NA31 Collaboration, *Phys. Lett.* B **317** (1993) 233; ibid. **206** (1988) 169.

[4] E731 Collaboration, *Phys. Rev. Lett.* **70** (1993) 1203.

[5] J. Kambor, J. Missimer and D. Wyler, *Nucl. Phys.* B **346** (1990) 17; *Phys. Lett.* B **261** (1991) 496.

[6] J. Bijnens, E. Pallante and J. Prades, *Nucl. Phys.* B **521** (1998) 305; E. Pallante, A. Pich and I. Scimemi, Nucl. Phys. B **617** (2001) 441; V. Cirigliano and E. Golowich, *Phys. Lett.* B **475** (2000) 351; *Phys. Rev.* D **65** (2002) 054014.

[7] V. Cirigliano *et al*, *Phys. Rev. Lett.* **91** (2003) 162001.

[8] E. Pallante and A. Pich, *Nucl. Phys.* B **592** (2001) 294; *Phys. Rev. Lett.* **84** (2000) 2568.

[9] A. Pich, these proceedings.

[10] J. Bijnens, E. Gámiz and J. Prades, *J. High Energy Phys.* **10** (2001) 009; hep-ph/0209089; hep-ph/0309216; V. Cirigliano *et al*, *Phys. Lett.* B **555** (2003) 71; S. Narison, *Nucl. Phys.* B **593** (2001) 3.

[11] J. Bijnens and J. Prades, *J. High Energy Phys.* **06** (2000) 035; hep-ph/0010008; hep-ph/0009155; hep-ph/0009156.

[12] S. Peris, M. Knecht and E. de Rafael, *Phys. Lett.* B **508** (2001) 117; S. Friot, D. Greynat and E. de Rafael, hep-ph/0408281; T. Hambye, S. Peris and E. de Rafael, *J. High Energy Phys.* **05** (2003) 027; T. Hambye *et al*, *Nucl. Phys.* B **564** (2000) 391; *Phys. Rev.* D **58** (1998) 014017.

[13] J.I. Noaki *et al.* (CP-PACS Collaboration), *Phys. Rev.* D **68** (2003) 014501; T. Blum *et al.* (RBC Collaboration), *Phys. Rev.* D **68** (2003) 114506; D. Bećirević *et al.* (SPQ$_{CD}$R Collaboration), *Nucl. Phys B (Proc. Suppl.)***119** (2003) 359.

[14] V. Kekelidze, these proceedings; R. Wanke, hep-ex/0305059; C. Cheshkov, hep-ex/0306012.

[15] A. Aloisio *et al.* [KLOE Collaboration], *Phys. Lett.* B **597** (2004) 139.

[16] V.F. Obraztsov and L.G. Landsberg, *Nucl. Phys B (Proc. Suppl.)* **99B** (2001) 257.

[17] I.V. Ajinenko *et al.*, *Phys. Lett.* B **567** (2003) 159.

[18] J. Bijnens, P. Dhonte and F. Persson, *Nucl. Phys.* B **648** (2003) 317.

[19] B.R. Holstein, *Phys. Rev.* **177** (1969) 2417; L.-F. Li and L. Wolfenstein, *Phys. Rev* D **21** (1980) 178; C. Avilez, *Phys. Rev.* D **23** (1981); B. Grinstein, S.-J. Rey and M.B. Wise, *Phys. Rev.* D **33** (1986) 1495; J.F. Donoghue, B.R. Holstein and G. Valencia, *Phys. Rev* D **36** (1987) 798; A.A. Bel'kov *et al.*, *Phys. Lett.* B **232** (1989) 118; *Int. J. Mod. Phys.* A **7** (1992) 1757; hep-ph/0311209; G. D'Ambrosio *et al.*, *Phys. Lett.* B **273** (1991) 497; *Phys. Rev.* D **50** (1994) 5767; Erratum ibid. **51** (1995) 3975; G. Isidori, L. Maiani and A. Pugliese, *Nucl. Phys.* B **381** (1992) 522; E.P. Shabalin, *Nucl. Phys.* B **409** (1993) 87; *Phys. Atom. Nucl.* **61** (1998) 1372; hep-ph/0305320.

[20] E. Gámiz, J. Prades and I. Scimemi, *J. High Energy Phys.* **03** (2003) 042; hep-ph/0305164; hep-ph/0405204.

[21] S. Weinberg, *Physica* A **96** (1979) 327; J. Gasser and H. Leutwyler, *Annals Phys.* **158** (1984) 142; *Nucl. Phys.* B **250** (1985) 465.

[22] G. Ecker, hep-ph/0011026; A. Pich, hep-ph/9806303.

[23] G. Ecker, *Prog. Part. Nucl. Phys.* **35** (1995) 1; E. de Rafael, hep-ph/9502254; A. Pich, *Rep. Prog. Phys.* **58** (1995) 563.

[24] J. Bijnens and F. Borg, *Nucl. Phys.* B **697** (2004) 319 ; F. Borg , hep-ph/0408350 ; A. Nehme , hep-ph/0406209.

[25] G. D'Ambrosio, G. Isidori and G. Martinelli, *Phys. Lett.* B **480** (2000) 164.